① $\lim_{x\to 0} \frac{\operatorname{sen} x}{x} = 1$

a) $\lim_{x\to 0} \frac{\operatorname{sen}(2x)}{x} = \frac{0}{0}$?

$\lim_{x\to 0} \frac{\operatorname{sen}(2x)}{x} = \frac{2\operatorname{sen}x \cdot \cos x}{x} =$

$= \frac{\cancel{\operatorname{sen}x} \cdot (2\cos x)}{\cancel{x}} = 2\cos x = 2$

$= \lim_{x\to 0} 2$

b) $\lim_{x\to 0} \frac{\operatorname{sen}(ax)}{\operatorname{sen}(bx)} = \frac{0}{0}$?

$\lim_{x\to 0} \frac{\operatorname{sen}(ax)}{\operatorname{sen}(bx)} \cdot \frac{\cancel{ax}}{\cancel{bx}} \cdot \frac{bx}{ax} = \lim_{x\to 0} \frac{a}{b}$

* Fazemos essas multiplicações para forçar que o limite conhecido apareça.

c) $\lim_{x\to 0} \frac{\operatorname{sen}3x}{\operatorname{sen}5x} = \frac{3}{5}$

d) $\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{0}{0}$?

$\lim_{x\to 0} \frac{(1-\cos x)}{x^2} \cdot \frac{(1+\cos x)}{(1+\cos x)} = \frac{1-\cos^2 x}{x^2 \cdot (1+\cos x)} =$

$= \frac{1-\cos^2 x}{x^2 \cdot (1+\cos x)} = \frac{\operatorname{sen}^2 x}{x^2 \cdot (1+\cos x)} = \left(\frac{\operatorname{sen}x}{x}\right)^2 \cdot \frac{1}{1+\cos x} =$

$= \lim_{x\to 0} \frac{1}{1+\cos x} = \lim_{x\to 0} \frac{1}{2}$

e) $\lim_{x\to 0} \frac{1-\cos x}{x} = \frac{1-1}{0} = \frac{0}{0}$?

$\lim_{x\to 0} \frac{(1-\cos x)}{x} \cdot \frac{(1+\cos x)}{(1+\cos x)} = \frac{1-\cos^2 x}{x \cdot (1+\cos x)} =$

$= \lim_{x\to 0} \frac{\operatorname{sen}^2 x}{x \cdot (1+\cos x)} = \frac{\operatorname{sen}x}{x} \cdot \frac{\operatorname{sen}x}{1+\cos x} =$

$= \lim_{x\to 0} \frac{\operatorname{sen}x}{1+\cos x} = \frac{0}{2} = 0$ //

f) $\lim_{x\to \pi} \frac{1-\operatorname{sen}\frac{x}{2}}{x-\pi} = \frac{1-1}{\pi-\pi} = \frac{0}{0}$?

$x-\pi \to 0$ $u = x-\pi$ $u\to 0$ $x = u+\pi$

$\lim_{x\to \pi} \frac{1-\operatorname{sen}\frac{x}{2}}{x-\pi} = \lim_{u\to 0} \frac{1-\operatorname{sen}\left(\frac{u}{2}+\frac{\pi}{2}\right)}{u} =$

$\operatorname{sen}\frac{u}{2} + \frac{\pi}{2} = \cos\frac{u}{2}$

$= \lim_{u\to 0} \frac{1-\cos\frac{u}{2}}{u} = \lim_{v\to 0} \frac{1-\cos v}{2v} \overset{*}{=}$

$v = \frac{u}{2} \to u = 2v$ $u\to 0$ $v\to 0$

$\overset{*}{=} \lim_{v\to 0} \frac{1-\cos v}{2v} = \lim_{v\to 0} \frac{1}{2} \cdot \left(\frac{1-\cos v}{v}\right) = 0$ //

g) $\lim_{x\to 0} \frac{1-\cos x}{x \cdot \operatorname{sen}x} = \frac{1-1}{0} = \frac{0}{0}$?

$\lim_{x\to 0} \frac{1-\cos x}{x \cdot \operatorname{sen}} \cdot \frac{1+\cos x}{1+\cos x} = \frac{1-\cos^2 x}{x \operatorname{sen}(1+\cos x)} =$

$= \frac{\operatorname{sen}^2 x}{x \cdot \operatorname{sen}(1+\cos x)} = \frac{\operatorname{sen}x}{x} \cdot \frac{1}{1+\cos x} = \lim_{x\to 0} \frac{1}{1+\cos x} = \frac{1}{2}$ //

h) $\lim_{x\to 0} \frac{1-\sec x}{x} = \frac{1-1}{0} = \frac{0}{0}$?

$\lim_{x\to 0} \frac{1-\sec x}{x} = \frac{1-\frac{1}{\cos x}}{x} = \frac{\frac{\cos x - 1}{\cos x}}{x} = \frac{\cos x - 1}{\cos x \cdot x}$

$\lim \frac{\cos x - 1}{\cos x \cdot x} \cdot \frac{\cos x + 1}{\cos x + 1} = \frac{\cos^2 x - 1}{\cos x \cdot (x+1)} = \frac{\cancel{\cos x} \cdot \left(\cos x - \frac{1}{\cos x}\right)}{\cancel{\cos x} \cdot (x+1)} = \lim_{x\to 0} \frac{\cos x - \frac{1}{\cos x}}{x+1} = \frac{1-1}{1} = 0$ //

i) $\lim_{x\to 0} \frac{\operatorname{tg} ax}{bx} = \frac{0}{0}$?

$ax = z$

$x = \frac{z}{a}$

$\lim_{x\to 0} \frac{\operatorname{tg} ax}{bx} = \lim_{x\to 0} \frac{\operatorname{tg} z}{b \cdot \frac{z}{a}} =$

$\lim_{x\to 0} \frac{\operatorname{tg} z}{b \cdot \frac{z}{a}} = \frac{z}{z} \cdot \frac{a}{b} =$

$= \lim_{x\to 0} \frac{a}{b}$ //

Temos que observar que quando $x\to 0$ $\operatorname{tg}(x) = x$

j) $\lim_{x\to 0} \frac{\operatorname{tg} 2x}{3x} = \frac{2}{3}$ //

k) $\lim_{x\to 0} \frac{\operatorname{tg}x + \operatorname{sen}x}{x} = \frac{0}{0} = ?$

$\lim_{x\to 0} \frac{\operatorname{tg}x + \operatorname{sen}x}{x} = \frac{\frac{\operatorname{sen}x}{\cos x} + \operatorname{sen}x}{x} =$

$\lim_{x\to 0} \frac{\operatorname{sen}\cdot(1+\cos)}{x \cdot \cos} = \frac{\operatorname{sen}x}{x} \cdot \frac{1+\cos x}{\cos x} =$

$= \lim_{x\to 0} \frac{1+\cos x}{\cos x} = 2$ //

l) $\lim_{x\to 1} \frac{1-x^2}{\operatorname{sen}(\pi x)} = \frac{0}{0}$?

$x-1\to 0$ $t = x-1$

$t\to 0$ $x = t+1$

$\lim_{x\to 1} \frac{1-x^2}{\operatorname{sen}(\pi x)} = \frac{(1-x)\cdot(1+x)}{\operatorname{sen}(\pi x)}$

$\lim_{t\to 0} \frac{(-t)\cdot(2+t)}{\operatorname{sen}(\pi t + \pi)} = \frac{-t\cdot(2+t)}{-\operatorname{sen}(\pi t)} = \frac{t}{\operatorname{sen}(\pi t)} \cdot \frac{(2+t)}{1} = \frac{2}{\pi}$ //

$\frac{t}{\operatorname{sen}(\pi t)} \cdot \frac{\pi t}{\pi t} = \frac{\pi t}{\operatorname{sen}(\pi t)} \cdot \frac{t}{\pi t} = \frac{1}{\pi} \cdot 1$ ou

$\frac{t\cdot(2+t)}{\operatorname{sen}(\pi t)} = \frac{2+t}{\frac{\operatorname{sen}(\pi t)}{t}} = \lim_{t\to 0} \frac{2}{\pi}$

$t\to 0$ lembrando que: $\lim_{x\to 0} \frac{\operatorname{sen}x}{x} = \frac{1}{1}$ e $\lim_{x\to 0} \frac{x}{\operatorname{sen}x} = \frac{1}{1}$

m) $\lim_{x\to 1} \frac{\cos\frac{\pi x}{2}}{1-x} = \frac{0}{0}$?

$\lim_{x\to 1} \frac{\cos\frac{\pi x}{2}}{1-x} = \frac{\operatorname{sen}\left(\frac{\pi}{2} - \frac{\pi x}{2}\right)}{1-x} =$

$= \lim_{x\to 1} \frac{\operatorname{sen}\left(\frac{\pi}{2}\cdot(1-x)\right)}{1-x} \overset{*}{=}$

$t = \frac{\pi}{2}\cdot(1-x)$

$(1-x) = \frac{2t}{\pi}$

$t\to 0$

$\overset{*}{=} \lim_{t\to 0} \frac{\operatorname{sen}t}{\frac{2t}{\pi}} = \lim_{t\to 0} \frac{\operatorname{sen}t}{t} \cdot \frac{\pi}{2} = \lim_{t\to 0} \frac{\pi}{2}$

* Deve-se observar com atenção a lógica por traz da comparação feita entre cos e sen e como está funcionando no limite com $x\to 1$ //

n) $\lim_{x\to 0} \frac{\sqrt{1+\operatorname{sen}x} - \sqrt{1-\operatorname{sen}x}}{x} = \frac{0}{0}$?

$a = \sqrt{1+\operatorname{sen}x}$

$b = \sqrt{1-\operatorname{sen}x}$

$\lim_{x\to 0} \frac{\sqrt{1+\operatorname{sen}x} - \sqrt{1-\operatorname{sen}x}}{x} = \frac{a-b}{x} \cdot \frac{a+b}{a+b} =$

$= \lim_{x\to 0} \frac{a^2-b^2}{x\cdot(a+b)} = \frac{1+\operatorname{sen}x - 1 + \operatorname{sen}x}{x\cdot(a+b)} = \frac{\operatorname{sen}x}{x} \cdot \frac{2}{a+b} =$

$= \lim_{x\to 0} \frac{2}{\sqrt{1+\operatorname{sen}x} + \sqrt{1-\operatorname{sen}x}} = \lim_{x\to 0} 1$ //

② a) $\lim_{x\to 0} \frac{\operatorname{sen}(x+0) - \operatorname{sen}0}{x} = \frac{\operatorname{sen}x\cdot\cos 0 + \operatorname{sen}0\cdot\cos x - \operatorname{sen}0}{x} = \frac{\operatorname{sen}x}{x} \cdot \left(\cos 0 + \frac{(\operatorname{sen}0\cdot\cos x - \operatorname{sen}0)}{\operatorname{sen}x}\right) = \cos 0$

algo muito próximo de 0

b) $\lim_{x\to 0} \frac{\cos(x+0) - \cos 0}{x} = \frac{\cos x\cdot\cos 0 - \operatorname{sen}x\cdot\operatorname{sen}0 - \cos 0}{x} = \frac{\cos 0\cdot(\cos x - 1) - \operatorname{sen}x\cdot\operatorname{sen}0}{x} = \frac{\operatorname{sen}x}{x} \cdot -\operatorname{sen}0 = -\operatorname{sen}0$ //

$\frac{\cos x - 1}{x} \neq 0$

quando $x\to 0$, mas não é zero!

$tg\,0 = 0 \quad x \to 0$

c) $\lim_{x\to 0} \frac{tg(x+0) - tg\,b}{x} = \lim_{x\to 0} \frac{\frac{tg\,x + tg\,0}{1 - tg\,x \cdot tg\,0} - tg\,b}{x} = \frac{tg\,x + tg\,0 - tg\,b\,(1 - tg\,x \cdot tg\,0)}{(1 - tg\,x \cdot tg\,0)\cdot x} = \frac{tg\,x\,(tg\,b \cdot tg\,0 + 1) + tg\,0 - tg\,b}{(1 - tg\,x \cdot tg\,0)\cdot x} = \lim_{x\to 0} \frac{tg\,0 - tg\,b}{x}$

d) $\lim_{x\to 0} \frac{\sec(x+0) - \sec 0}{x} = \frac{\frac{1}{\cos(x+0)} - \frac{1}{\cos 0}}{x} = \frac{\cos 0 - \cos(x+0)}{\cos(x+0)\cdot\cos 0 \cdot x} = \frac{\cos 0 - \cos(x+0)}{x} \cdot \frac{1}{\cos 0 \cdot \cos(x+0)} = \frac{\cos 0 - (\cos x \cdot \cos 0 - \operatorname{sen} x \cdot \operatorname{sen} 0)}{x} \cdot \frac{1}{\cos 0 \cdot \cos(x+0)} = \text{I}$

I $\frac{\cos 0 - (\cos x \cdot \cos 0 - \operatorname{sen} x \cdot \operatorname{sen} 0)}{x} \cdot \frac{1}{\cos 0 \cdot \cos(x+0)} = \left[\frac{\cos 0 \cdot (-\cos x + 1)}{x} + \frac{\operatorname{sen} x \cdot \operatorname{sen} 0}{x}\right] \cdot \frac{1}{\cos 0 \cdot \cos(x+0)} = \frac{\operatorname{sen} 0}{\cos 0 \cdot \cos(x+0)} = \frac{\operatorname{sen} 0}{\cos 0} \cdot \frac{1}{\cos 0} = tg\,0 \cdot \sec 0$ //

$x \to 0$; 0; 1

3 a) $\lim_{x\to\infty} \ln(x) = \log_e \infty = +\infty$

b) $\lim_{x\to 0^+} \ln(x) = -\infty$

* cada vez que se aproxima de zero, fica mais negativo

$\log_e x = y \to e^y = x$ → é dizer claro que estamos falando

c) $\lim_{x\to 3} \log\left|\frac{6x+2}{4x+3}\right| \to \log \frac{4}{3}$

d) $\lim_{x\to 0} \log \frac{x - x^3}{x^2 + x} = \frac{0}{0}$?

$\lim_{x\to 0} \log \frac{x - x^3}{x^2 + x} = \frac{\cancel{x}\cdot(1 - x^2)}{\cancel{x}\cdot(x+1)}$

$\lim_{x\to 0} \log \frac{1 - x^2}{x+1} = \log \frac{1}{1} = 0$ //

e) $\lim_{x\to -1} \log_3 \frac{x^2 + 3x + 2}{x^2 + 5x + 4} = \log_3 \frac{0}{0}$?

$\lim_{x\to -1} \log_3 \frac{x^2 + 3x + 2}{x^2 + 5x + 4} = \log_3 \frac{\cancel{(x+1)}\cdot(x+2)}{\cancel{(x+1)}\cdot(x+4)}$

$\lim_{x\to -1} \log_3 \frac{(x+2)}{(x+4)} = \log_3 \frac{1}{3} = -1$

f) $\lim_{x\to 3} \ln \frac{x-3}{\sqrt{x+7} - 2} = \frac{0}{0}$?

$\lim_{x\to 3} \ln \frac{x-3}{\sqrt{x+7} - 2} = \ln \frac{x-3}{\sqrt{x+7} - 2} \cdot \frac{\sqrt{x+7} + 2}{\sqrt{x+7} + 2} = \ln \frac{(x-3)\cdot(\sqrt{x+7} + 2)}{x-3} = \ln \sqrt{x+7} + 2 = \lim_{x\to 3} \ln 4$ //

4 a) $\lim_{x\to\infty} \left[\left(1 + \frac{1}{x}\right)^x\right]^2 = \lim_{x\to\infty} e^2$ //

b) $\lim_{x\to\infty} \left(1 + \frac{2}{x}\right)^x = 1^\infty$?

$\frac{1}{t} = \frac{2}{x} \Rightarrow t = \frac{x}{2}$

$x \to \infty \quad t \to \infty$

$\lim_{t\to\infty} \left(1 + \frac{1}{t}\right)^{2t} = \lim_{t\to\infty} \left[\left(1 + \frac{1}{t}\right)^t\right]^2 = e^2$

d) $\lim_{x\to 0} (1 + x)^{\frac{1}{x}}$ $\quad \frac{1}{x} = t \quad x = \frac{1}{t}$

$t \to \infty$

$\lim_{t\to\infty} \left(1 + \frac{1}{t}\right)^t = e$ // * Se é $\frac{1}{x}$ e $x \to 0$, então $t \to \infty$

e) $\lim_{x\to -\infty} \left(1 + \frac{4}{x}\right)^{\frac{3x}{5}} \to \lim_{x\to -\infty} \left[\left(1 + \frac{4}{x}\right)^x\right]^{\frac{3}{5}} = \left(e^4\right)^{\frac{3}{5}} = e^{\frac{12}{5}}$ * negativo, pois $x \to -\infty$

f) $\lim_{x\to\infty} \left(\frac{x}{x+1}\right)^x = \left[\left(\frac{x+1}{x}\right)^x\right]^{-1} = \lim_{x\to\infty} \frac{1}{e}$

g) $\lim_{x\to\infty} \left(\frac{x-1}{x+1}\right)^x = \frac{\lim_{x\to\infty} (x-1)^x}{\lim_{x\to\infty} (x+1)^x} = \frac{\lim_{x\to\infty} \cancel{x}\cdot\left(1 - \frac{1}{x}\right)^x}{\lim_{x\to\infty} \cancel{x}\cdot\left(1 + \frac{1}{x}\right)^x} = \frac{\lim_{x\to\infty} \left[\left(1 + \frac{1}{t}\right)^t\right]^{-1}}{\lim_{x\to\infty} e} = \frac{\lim_{x\to\infty} \frac{1}{e}}{\lim_{x\to\infty} e} = \lim_{x\to\infty} \frac{e^{-1}}{e} = e^{-2}$ //

$e^{-1-(1)} = e^{-2}$

$\frac{1}{x} = -\frac{1}{t} \quad t = -x \quad x = -t$

h) $\lim_{x\to\infty} \left(\frac{x-4}{x-1}\right)^{x+3} = \left[\frac{\cancel{x}\cdot\left(1 - \frac{4}{x}\right)}{\cancel{x}\cdot\left(1 - \frac{1}{x}\right)}\right]^{x+3} = \left[\frac{1 - \frac{4}{x}}{1 - \frac{1}{x}}\right]^x \cdot \left[\frac{1 - \frac{4}{x}}{1 - \frac{1}{x}}\right]^3 = \frac{\lim_{x\to\infty} \left(1 - \frac{4}{x}\right)^x}{\lim_{x\to\infty} \left(1 - \frac{1}{x}\right)^x} \cdot \frac{\lim_{x\to\infty} \left(1 - \frac{4}{x}\right)^3}{\lim_{x\to\infty} \left(1 - \frac{1}{x}\right)^3} = \frac{\lim_{x\to\infty} \left[\left(1 + \frac{1}{t}\right)^t\right]^{-4}}{\lim_{x\to\infty} \left[\left(1 + \frac{1}{m}\right)^m\right]^{-1}} \cdot = \lim_{x\to} e^{-3}$ //

$\to 1$; $\to e^{-4}$; $\to e^{-1}$

$\frac{-4}{x} = \frac{1}{t} \quad x = -4t \quad \frac{-1}{x} = \frac{1}{m} \quad x = -m$

i) $\lim_{x\to\infty} \left(\frac{x-3}{x+2}\right)^x = \left[\frac{\cancel{x}\cdot\left(1 - \frac{3}{x}\right)}{\cancel{x}\cdot\left(1 + \frac{2}{x}\right)}\right]^x = \frac{\lim_{x\to\infty} \left(1 - \frac{3}{x}\right)^x}{\lim_{x\to\infty} \left(1 + \frac{2}{x}\right)^x} = \frac{\lim_{x\to\infty} \left[\left(1 + \frac{3}{3t}\right)^t\right]^{-3}}{\lim_{x\to\infty} \left[\left(1 + \frac{2}{2m}\right)^m\right]^2} = \lim_{x\to\infty} \frac{e^{-3}}{e^2} = \lim_{x\to\infty} e^{-5}$ //

$\frac{-3}{x} = \frac{1}{t} \quad t = -3x \quad \frac{2}{x} = \frac{1}{m} \quad x = 2m$

j) $\lim_{x\to\infty} \left(\frac{2x+3}{2x+1}\right)^x = \left[\frac{\cancel{x}\cdot\left(2 + \frac{3}{x}\right)}{\cancel{x}\cdot\left(2 + \frac{1}{x}\right)}\right]^x = \lim_{x\to\infty} \left(\frac{\cancel{2}\cdot\left(1 + \frac{3}{2x}\right)}{\cancel{2}\cdot\left(1 + \frac{1}{2x}\right)}\right)^x = \frac{\lim_{x\to\infty} \left(1 + \frac{3}{2x}\right)^x}{\lim_{x\to\infty} \left(1 + \frac{1}{2x}\right)^x} = \frac{\lim_{t\to\infty} \left[\left(1 + \frac{1}{t}\right)^t\right]^{\frac{3}{2}}}{\lim_{m\to\infty} \left[\left(1 + \frac{1}{m}\right)^m\right]^{\frac{1}{2}}} = \frac{e^{\frac{3}{2}}}{e^{\frac{1}{2}}} = e^{\frac{3}{2} - \frac{1}{2}} = \lim_{x\to\infty} e$

$\to e$

$\frac{3}{2x} = \frac{1}{t} \to x = \frac{3t}{2} \quad \frac{1}{2x} = \frac{1}{m} \to m = 2x \quad x = \frac{m}{2}$

k) $\lim_{x\to -\infty} \left(\frac{2x-1}{2x+1}\right)^x = \left[\frac{\cancel{x}\cdot\left(2 - \frac{1}{x}\right)}{\cancel{x}\cdot\left(2 + \frac{1}{x}\right)}\right]^x \to \left[\frac{\cancel{2}\cdot\left(1 - \frac{1}{2x}\right)}{\cancel{2}\cdot\left(1 + \frac{1}{2x}\right)}\right]^x = \frac{\lim_{x\to -\infty} \left(1 - \frac{1}{2x}\right)^x}{\lim_{x\to -\infty} \left(1 + \frac{1}{2x}\right)^x} = \frac{\lim_{t\to\infty} \left(\left(1 + \frac{1}{t}\right)^t\right)^{-\frac{1}{2}}}{\lim_{m\to\infty} \left(\left(1 + \frac{1}{m}\right)^m\right)^{\frac{1}{2}}} = \frac{e^{-\frac{1}{2}}}{e^{\frac{1}{2}}} = e^{-\frac{1}{2} - \frac{1}{2}} = \frac{1}{e}$

$-\frac{1}{2x} = \frac{1}{t} \to t = -2x \quad x = -\frac{t}{2} \quad \frac{1}{2x} = \frac{1}{m} = m = 2x \quad x = \frac{m}{2}$

$$l)\ \lim_{x\to-\infty}\left(\frac{3x+2}{3x-1}\right)^{2x} = \left(\frac{\cancel{x}\cdot\left(3+\frac{2}{x}\right)}{\cancel{x}\cdot\left(3-\frac{1}{x}\right)}\right)^{2x} = \frac{\lim_{x\to-\infty} \cancel{3}\cdot\left(1+\frac{2}{3x}\right)}{\lim_{x\to-\infty} \cancel{3}\cdot\left(1-\frac{1}{3x}\right)} = \frac{\lim_{t\to\infty}\left[\left(1+\frac{1}{t}\right)^{t}\right]^{4/3}}{\lim_{m\to\infty}\left[\left(1+\frac{1}{m}\right)^{m}\right]^{-2/3}} = \frac{e^{4/3}}{e^{-2/3}} = e$$

$$\frac{1}{t} = \frac{2}{3x} \quad x = \frac{2t}{3} \qquad \frac{1}{m} = -\frac{1}{3x} \quad x = -\frac{m}{3}$$

5) $$\lim_{x\to0}\frac{a^x-1}{x} = \ln(a)$$

$$a)\ \lim_{x\to0}\frac{e^{2x}-1}{x} = \lim_{x\to0}\frac{e^{u}-1}{\frac{u}{2}} = 2\ln e = 2 \qquad 2x = u \quad x = \frac{u}{2}$$

$$b)\ \lim_{x\to0}\frac{2^{3x}-1}{x} = \lim_{x\to0}\frac{2^{u}-1}{u}\cdot\frac{3}{1} = 3\cdot\ln 2 \qquad 3x = u \quad x = \frac{u}{3}$$

$$c)\ \lim_{x\to0}\frac{3^{2x}-1}{2^{5x}-1} = \frac{\left(e^{2x\ln3}-1\right)}{\left(e^{5x\ln2}-1\right)}\cdot\frac{\frac{2x\ln3}{2x\ln3}}{\frac{5x\ln2}{5x\ln2}} = \frac{\frac{\left(e^{2x\ln3}-1\right)}{2x\ln3}\cdot\frac{2x\ln3}{1}}{\frac{\left(e^{5x\ln2}-1\right)}{5x\ln2}\cdot\frac{5x\ln2}{1}} = \frac{\cancel{\ln e}}{\cancel{\ln e}}\cdot\frac{2\cancel{x}\ln3}{5\cancel{x}\ln2} = \frac{2\cdot\ln3}{5\cdot\ln2}$$

$$2^{5x}-1 = \left(e^{5x\ln2}-1\right)$$

$$3^{2x}-1 = \left(e^{2x\ln3}-1\right)$$

$$\frac{\frac{2x\ln3}{2x\ln3}}{\frac{5x\ln2}{5x\ln2}} = 1$$

$$d)\ \lim_{x\to0}\frac{e^{2x}-1}{e^{3x}-1} = \frac{e^{2x}-1}{e^{3x}-1}\cdot\frac{\frac{2x}{2x}}{\frac{3x}{3x}} = \frac{\frac{e^{2x}-1}{2x}\cdot\frac{2x}{1}}{\frac{e^{3x}-1}{3x}\cdot\frac{3x}{1}} = \frac{\cancel{\ln e}}{\cancel{\ln e}}\cdot\frac{2\cancel{x}}{3\cancel{x}} = \frac{2}{3}$$

$$e)\ \lim_{x\to2}\frac{e^{x}-e^{2}}{x-2} = \frac{e^{x-2}-e^{2}}{x-2} = \frac{e^{m+2}-e^{2}}{m} = \frac{e^{m}\cdot e^{2}-e^{2}}{m} = \frac{e^{2}}{1}\cdot\frac{\left(e^{m}-1\right)}{m} = e^{2}$$

$\ln e$

$m = x-2 \qquad x = m+2$

* A forma indeterminada só aparece quando $a = 2$

$$f)\ \lim_{x\to2}\frac{2^{x}-2^{0}}{x-0} \to \frac{2^{x}-4}{x-2} = \frac{e^{x\cdot\ln2}-e^{2\ln2}}{x-2} = \frac{e^{2\ln2}\cdot\left(e^{(x-2)\cdot\ln2}-1\right)}{x-2} = \frac{4\cdot\left(e^{(x-2)\cdot\ln2}-1\right)}{x-2} = \lim_{t\to0}\frac{4\cdot\left(e^{t}-1\right)}{\frac{t}{\ln2}} = \lim_{t\to0}4\ln2\cdot\left(\frac{e^{t}-1}{t}\right) = 4\ln2$$

$\ln e = 1$

$t = (x-2)\ln2 \qquad (x-2) = \frac{t}{\ln2} \qquad x-2\to0 \quad t\to0$

$$g)\ \lim_{x\to0}\frac{e^{x}-e^{0}}{x-0} = \lim_{t\to0}\frac{e^{(t+0)}-e^{0}}{t} = \frac{e^{t}\cdot e^{0}-e^{0}}{t} = \frac{e^{0}\cdot\left(e^{t}-1\right)}{t} = e^{a}$$

$\ln e = 1$

$t = x-0$

$x = t+0$